

Coupled Climate-Carbon Cycle Modeling And Spinning Up the Ocean Model: A Journey

Keith Lindsay

NCAR/CGD

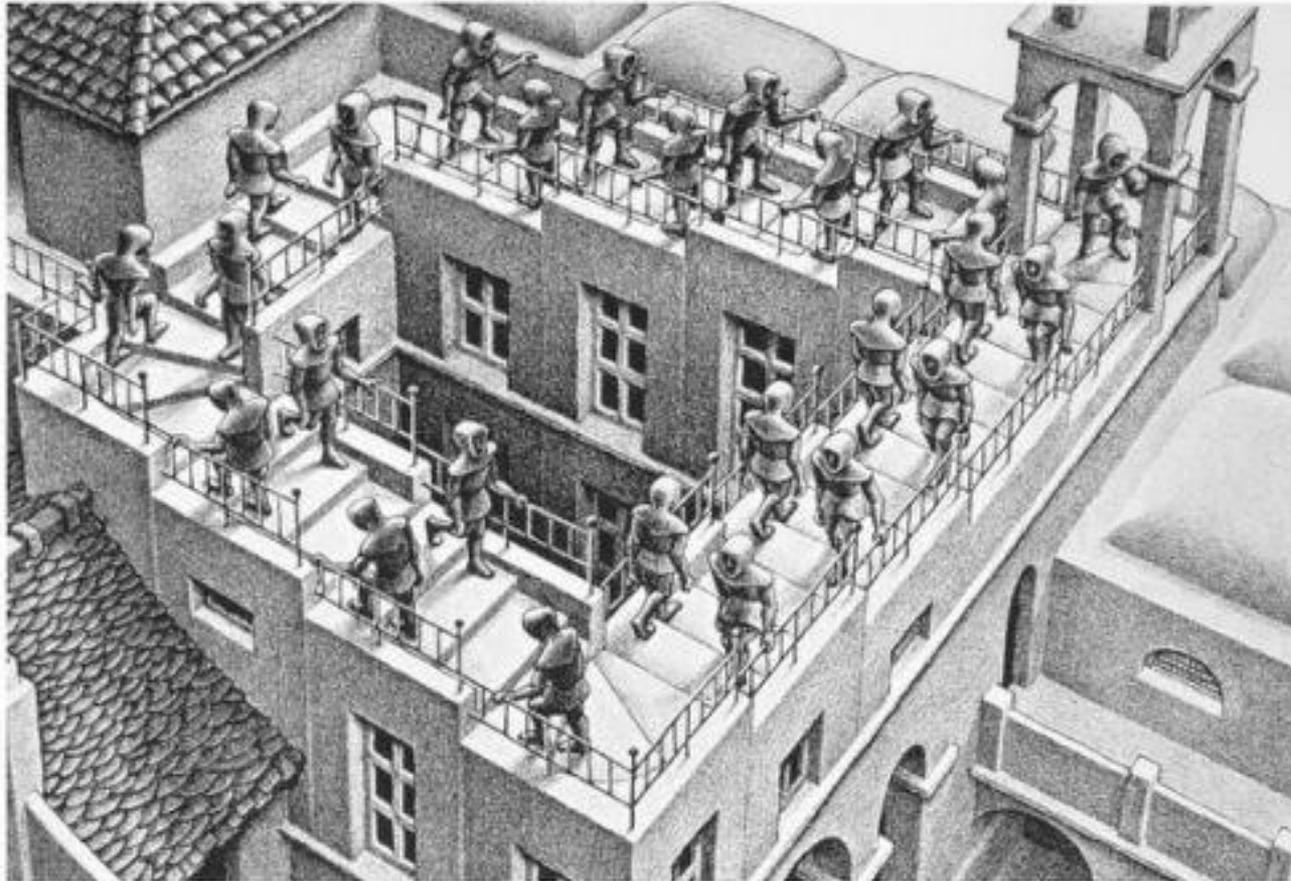
June 17, 2019



The journey is not solo



**It doesn't always seem like
you're moving forward**



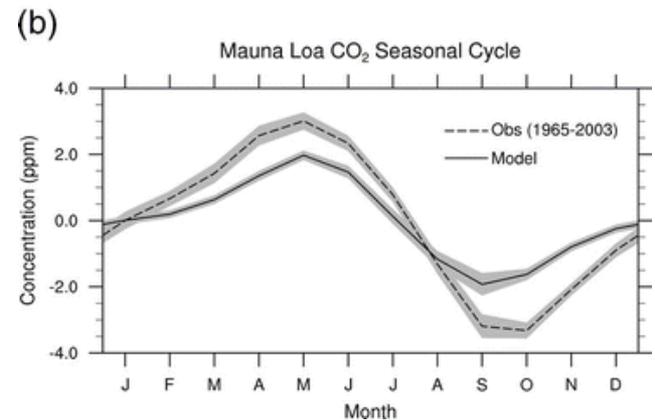
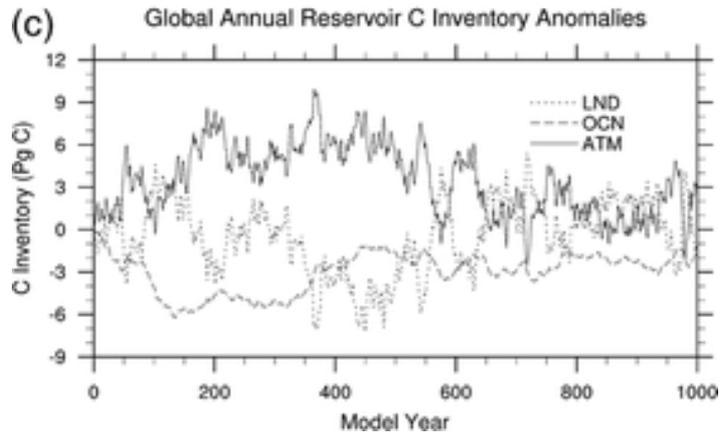
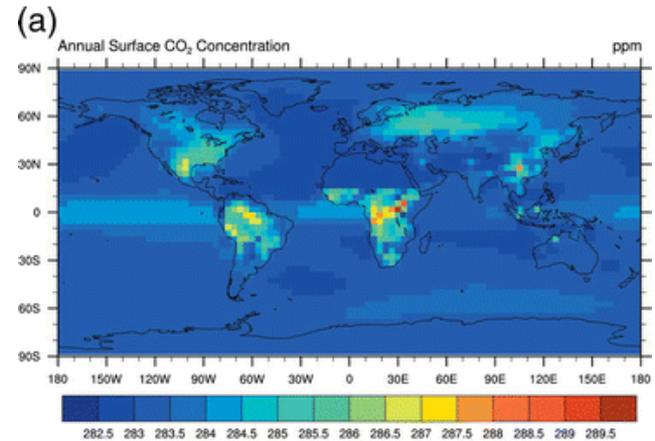
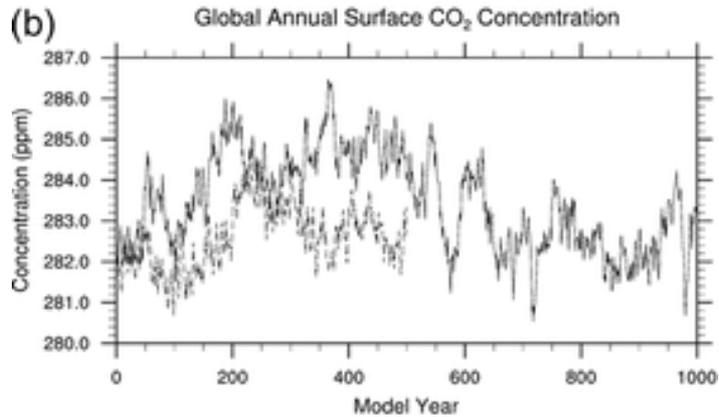
Coupled Climate-Carbon Cycle Modeling with CSM1.4, aka Flying Leap (c. 2000)

- Idea: take land and ocean carbon cycle models, couple them to atmospheric CO₂, and see what happens
- Land: add modified CASA BGC model to CSM's Land Surface Model (LSM)
- Ocean: develop and add low complexity BGC model to CSM's NCAR Ocean Model (NCOM)
- Add CO₂ tracers to atmospheric model
- Spin-up, couple, and leap

Synopsis of CSM1.4 BGC Spin-ups

- Land and ocean carbon pools need to be spun-up in order to have stable atmospheric CO₂
- BGC resolution (atm, ocn): (T31, x3) \approx (3.75°, 3.6°)
 - non-BGC resolution: (T42, x2) \approx (2.8°, 2.4°)
- Spinup land model for ~1000 years
- Spinup ocean model for ~350 surface years
- Use depth-dependent acceleration for ocean spin-up
 - Danabasoglu, McWilliams, Large, *J. Clim.*, 1996
 - Timesteps at depth 50x longer than at surface
 - Ad hoc fixers to correct for non-conservation

Flying Leap CSM1.4 Results



Doney, Lindsay, Fung, John, *J. Clim.*, 2006

CSM (renamed CCSM) wasn't standing still

- CCSM2 was released in May 2002
 - Included the transition to the ocean model POP
 - Ocean resolution refined from x2 $\approx 2.4^\circ$ to gx1 $\approx 1.1^\circ$
- CCSM3 was released in June 2004
 - Documented in 2006 *J. Clim.* Special Issue
 - Included refinement of atmospheric resolution to T85 $\approx 1.4^\circ$



Coupled Climate-Carbon Cycle Modeling with CCSM3.1

- Nitrogen cycle added to land BGC
 - Thornton et al., *GBC*, 2007
- Explicit ecosystem added to ocean BGC
 - Moore, Doney, Lindsay, *GBC*, 2004
- Ocean spun-up for 1000 years, no acceleration
 - not clear how to handle non-conservation with full ecosystem
- Resolution (T31, gx3) $\approx (3.75^\circ, 3.6^\circ)$
 - Yeager et al., *J. Clim.*, 2006
- Thornton et al., *Biogeosciences*, 2009

Looking towards CCSM4/CESM1 (ultimately released in 2010)

- Goal: include BGC in supported release at same resolution as physics
 - POP1 → POP2, 40 layers → 60 layers
- Ocean BGC status to SSC
 - June 2008: “major issue for coupled spin-up is the slow equilibration time-scale for ocean CO₂ system”
 - November 2008: “still working on spin-up techniques; may have to use brute force spin-up & live with some drift”
- Newton-Krylov solvers emerged in the literature
 - Li, Primeau, *Ocean Model.*, 2008
 - Khatiwala, *Ocean Model.*, 2008

Newton-Krylov solvers (not Newton vs. Krylov)

Nikita Krylov (21-5) vs. Emanuel Newton (26-12-1) set for #EFN67 on 6/2.



Statement of Ocean Spin-up Problem

- Generate tracer distributions that are in balance with respect to (non-stationary) ocean model circulation.
- Applications:
 - Initializing transient experiments
 - Analyze dynamics/properties of spun-up tracers
 - Compare tracers to observations
 - Optimize parameters to reduce model bias
 - Requires ability to spin-up repeatedly
- Brute force is prohibitively expensive
 - wall-clock time and computing allocation
 - $(2000 \text{ yrs}) / (50 \text{ yrs/day}) = 40 \text{ days}$



Mathematical Formulation of Problem

- Let $c(t)$ denote tracer state, i.e., tracer concentrations.
 - for 1 tracer on $gx1$ grid, c is vector of length $\approx 4.2 \times 10^6$
- Model Map: $c(t) = \Phi(c(0), t)$
- Φ incorporates advection, mixing, surface fluxes, interior source-sink terms, etc.

- Find c^* such that $\Phi(c^*, T) = c^*$.
 - Tracer end-state is the same as the initial condition.
 - T is period of forcing and circulation.

- Rewrite as $G(c) \equiv \Phi(c, T) - c = 0$.



Newton's Method

- Iterative method for solving $G(c) \equiv \Phi(c, T) - c = 0$
- Generate sequence $c_1, c_2, \dots, c_k, \dots$ that converge to solution of system of equations
- $0 = G(c_{k+1}) = G(c_k) + (\partial G / \partial c) * (c_{k+1} - c_k) + \dots$

$$c_{k+1} = c_k - (\partial G / \partial c)^{-1} * G(c_k)$$

Computing the Increment in Newton's Method

- We cannot compute, or store, $(\partial G/\partial c)$
 - It is a dense matrix of size $4.2 \times 10^6 \times 4.2 \times 10^6$.
- We can evaluate matrix-vector products such as $(\partial G/\partial c)(\delta c)$ with the finite difference approximation

$$(\partial G/\partial c)(\delta c) \approx (G(c+\sigma\delta c) - G(c)) / \sigma$$

- Note this is a forward model run of length T .
- Krylov iterative methods are well suited for this scenario.

Krylov Methods

- Use Krylov iterative method (GMRES) to solve:

$$(\partial G/\partial c)(\delta c_k) = -G(c_k)$$

- Each GMRES iteration evaluates $(\partial G/\partial c)(\delta c)$
 - Note this is a forward model run of length T.

- Construct Krylov basis

$$y_0, (\partial G/\partial c)y_0, (\partial G/\partial c)^2 y_0, (\partial G/\partial c)^3 y_0, \dots$$

- Find linear combination of basis that minimizes

$$|(\partial G/\partial c) x + G(c_k)|^2$$

Preconditioner for GMRES

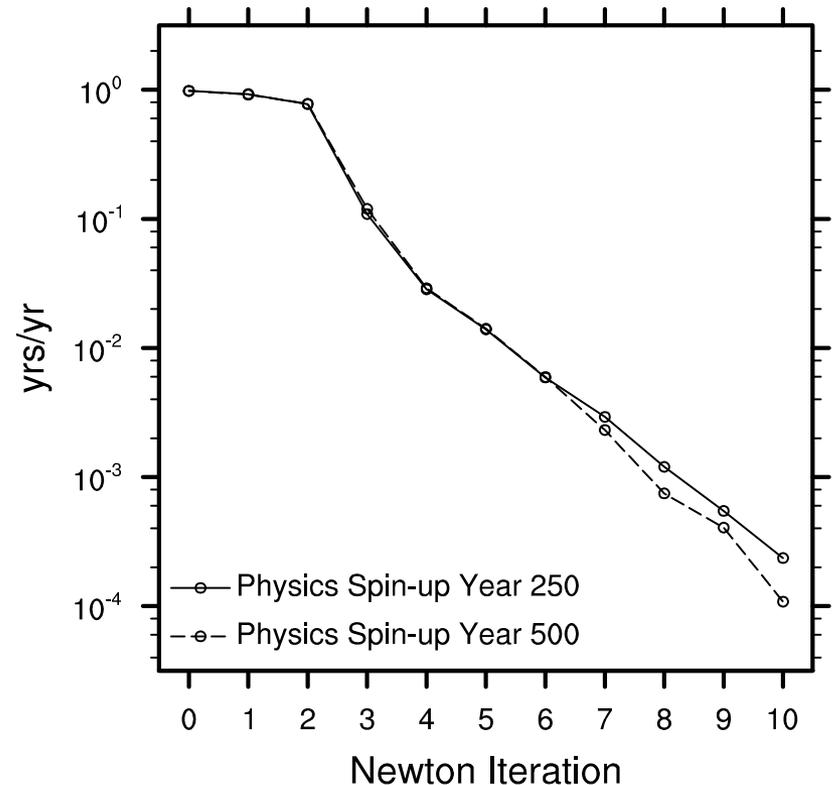
- Apply a preconditioner to GMRES, transforming
$$(\partial G/\partial c)(\delta c_k) = -G(c_k)$$
 into
$$(P(\partial G/\partial c))(\delta c_k) = (P)(-G(c_k))$$
- To improve convergence, $P \approx (\partial G/\partial c)^{-1}$.
- To be practical, multiplying by P should be feasible.
- We use a preconditioner specific to ocean circulation, and implement multiplication by P with a parallel sparse matrix solver, SuperLU.

Putting it all together

- Find c^* such that $G(c^*) \equiv \Phi(c^*, T) - c^* = 0$.
 - Tracer end-state is the same as the initial condition.
- Use Newton's method, $c_{k+1} = c_k - (\partial G / \partial c)^{-1} * G(c_k)$
- Use Krylov method (GMRES) to solve for δc_k
 - matrix-vector multiplies in GMRES are approximated with a model run
- Numerous technical details omitted
 - Construction and application of preconditioner.
 - Scripting challenges running CESM and processing results inside nested iterative solvers

Convergence vs Newton Iteration

- Ideal Age tracer (IAGE)
- Resolution $gx1 \approx 1.1^\circ$
- 60 layers
- Forced simulations
- Global RMS change in IAGE over 1 year
- Faster than brute force by a factor of >200



Lindsay, *Ocean Model.*, 2017

Newton-Krylov Solver Progress Summary

- 2009: Success for Ideal Age (IAGE) at gx3 resolution
- 2010: Modest success for BGC at gx3 resolution
 - Unable to extend to gx1 resolution, too late for CESM1
- 2013: Success for Ideal Age at gx1 resolution
 - Unable to translate success to BGC at gx1
 - BGC updates in CESM1.2 broke the BGC success at gx3
- Became clear no full success for BGC at gx1 for CESM2
- Other successes along the way:
 - Abiotic ^{14}C in the LGM
 - Noble gases (Ne, Ar, Kr, ...)
 - ^3He , ^4He isotopes

BGC at gx1 for CESM2

- 2017: Idea: target subset of BGC tracers
- Because we couple carbon, it is most important to have it spun-up
- Carbon is easier than nutrients and O_2
- But not necessarily easy
 - Nonlinear burial of $CaCO_3$ at seafloor
- 2018: NK generated IC for CESM2 for carbon
- Drift in NK based spin-up was 0.02 Pg/yr
- Drift in piControl is 0.04 Pg/yr

The journey continues

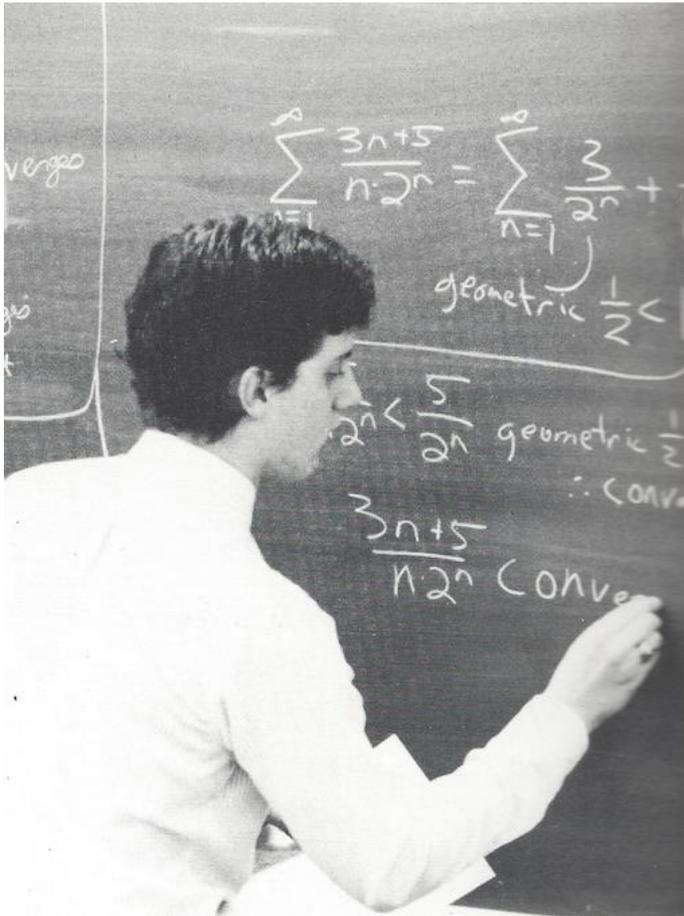
- Why did drift increase from spin-up to piControl?
- Continue efforts applying NK solver to nutrients and O_2
 - Use shadow tracer framework to spin-up nutrients with fixed quasi-spun-up productivity field
- Extend shadow tracer approach to active tracers
- Interannual variability: How many years of circulation are enough to be representative?
- Transition to MOM6



Extra Slides



Math has always played a role in my journey



Hilbert

Courant

Friedrichs

Lax

Chorin

Krasny

Lindsay

Are you sure that you have the right Keith?



Keith Moore



Keith Oleson



Keith Rodgers



Keith Lindsay



Lindsay Keith